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EFFECTS OF PROOF-TEST ON THE STRENGTH AND FATIGUE LIFE OF A UNI--ETC(U)

MAY 79 A S WANG , P C CHOU , J ALPER

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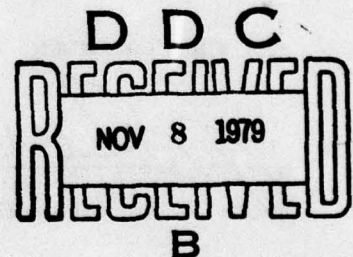
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**EFFECTS OF PROOF-TEST ON THE  
STRENGTH AND FATIGUE LIFE OF A  
UNIDIRECTIONAL COMPOSITE**

*DYNA EAST CORPORATION  
227 HEMLOCK ROAD  
WYNNEWOOD, PENNSYLVANIA 19096*

MAY 1979



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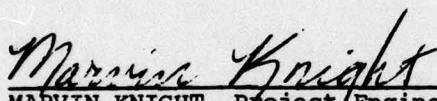
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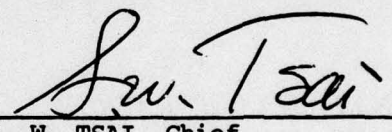
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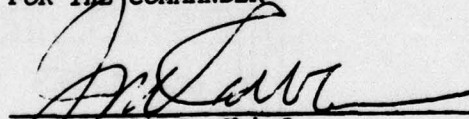
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MARVIN KNIGHT, Project Engineer  
Mechanics & Surface Interactions Br.  
Nonmetallic Materials Division

  
S. W. TSAI, Chief  
Mechanics & Surface Interactions Br.  
Nonmetallic Materials Division

FOR THE COMMANDER

  
J. M. KELBLE, Chief  
Nonmetallic Materials Division

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## Foreword

This is an interim report of a program sponsored by Air Force Materials Laboratory, Air Force System Command, Wright-Patterson Air Force Base, Ohio 45433, under contract F33615-77-C-5039 with Dyna East Corporation. The Air Force Project monitor is Mr. Marvin Knight. Dr. Pei Chi Chou is the principal investigator. He is assisted by Dr. Robert Croman in the theoretical analysis. The experimental phase of the research is carried out by Dr. A.S.D. Wang, with the assistance of Mr. James Alper.

The project is for a duration of 30 months. This report covers work performed during the period April 1, 1978 to March 31, 1979

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## SUMMARY

The effects of a static proof-load on the statistical distribution of the static strength and fatigue life of a unidirectional graphite-epoxy laminate (as-3501-05) are investigated experimentally. Loading mode for both the static and fatigue tests is restricted to uniaxial tension in the fiber direction. 6-ply tension coupons with dimensions of 22.9 cm x 1.9 cm are used; and all tests are conducted using a closed-loop Instron tester, under room temperature ( $\sim 21^\circ$ ) and ambient humidity ( $\sim 60\%$  R.H.) conditions. The test data are analyzed using a two-parameter Weibull distribution in conjunction with the maximum likelihood method for parameter estimation. Results show that proof-load can guarantee a minimum static strength, and to a less degree can assure a minimum fatigue life.

## SECTION I

### INTRODUCTION

In recent years, there has been a considerable amount of interest in the fatigue damage states in composite laminates. The problem of post-fatigue, residual strength and life in laminates attracted most attention. Several statistical methods are available to predict the residual strength, or life, based on some fatigue degradation laws [1-5]. Essentially, it is assumed that a unique relationship exists between the static strength and fatigue life; and the predictive model describes mathematically this relationship. For a specimen of a given population, the assumption stipulates that its rank in static strength shall be the same as its rank in fatigue life. This is known as the "equal rank" assumption.

In an effort to verify experimentally the "equal-rank" assumption, Hahn and Kim [2] employed the concept of proof-test and studied the effects on static-fatigue (creep) behavior of unidirectional glass-epoxy laminates. Later, Awerbuch and Hahn [6] conducted proof-tests on unidirectional graphite-epoxy laminates and examined the effects on fatigue behavior under cyclic load. In both experiments, test data in static strength and fatigue life seemed to confirm the "equal-rank" assumption. It was also observed that a minimum life can be guaranteed for specimens that survive the proof-test, if the proof-load does not cause appreciable damage, or degradate the fatigue properties. The extent of this requirement needs further study, however.

The main objective of the present paper is to investigate the effects of proof-load on both the (post-proof) static strength behavior and fatigue life behavior of unidirectional graphite-epoxy laminates. It enlarges the data-base which is still deemed insufficient. From the present test results, it is found that proof-loading does not change the essential features in the static strength;



and the procedure removes the weaker specimens from the population, thus guaranteeing a minimum strength for the specimens that survive the proof-test. Moreover, proof-loading degrades only slightly the fatigue property of the specimens; hence the procedure can still guarantee a minimum life, with a high degree of confidence. In the present experiment, the AS-3501-05 graphite-epoxy system is used. All tests are conducted under room temperature and ambient humidity (about 60% R.H.) conditions. The overall test program consists of 8 different tests, and a total of 304 data points are obtained.

## SECTION II

### EXPERIMENT

The present experimental program includes the following tests: (1) baseline data generation. This consists of static tension to failure, fatigue-life at a constant maximum load of 71% of the static mean strength ( $0.71 S_m$ ) and fatigue-life at  $0.81 S_m$  maximum fatigue load; (2) post-proof static strength study. This consists of proof-loading the specimens to  $88\% S_m$  and  $95\% S_m$ , respectively, followed by static tension to failure; and (3) post-proof fatigue-life study. Here, two tests are conducted for specimens proof-loaded to  $88\% S_m$  and then subjected to fatigue load to failure at  $71\% S_m$  and  $81\% S_m$  respectively; and one test for specimens proof-loaded to  $95\% S_m$  and then subjected to fatigue load at  $81\% S_m$ . The number of specimens tested in each case is listed in TABLE I.

#### Specimens

The material system used for this study is the AS-3501-05, with a nominal fiber content of 65% by volume. Test specimens are cut from panels supplied from the manufacturer directly. The dimensions of the test specimens are 6-ply thick (0.084 cm), 1.9 cm wide and 22.9 cm long with glass-epoxy end-tabs of 3.8 cm in length. Thus, the test gage length is 15.3 cm.

### Static Tension Test

The static tension tests are conducted on a closed-loop Instron tester under room temperature ( $\sim 21^{\circ}\text{C}$ ) and ambient humidity ( $\sim 0.6$  R.H.) conditions. The loading rate selected is approximately 4000 lb/min (1800 kg/min). The selection of test specimens follows a random number schedule.

### Fatigue Life Test

The fatigue tests are also conducted on the Instron tester under the same room temperature and ambient humidity condition. No effort is made to monitor any temperature change in the specimen during fatigue. The loading procedure is as follows: the specimen is first loaded statically with manual control to the mean stress level; it is then subjected to oscillatory loading with the maximum to minimum stress ratio of  $R = 0.1$ . The running cyclic frequency is 9.5 Hz. Most tests are carried to fatigue failure; some are suspended for purposes of either residual strength measurement, or the reduction of testing time.

### Proof-Test

(1) Post-proof tensile strength. This part of the proof-test consists of loading the specimens statically to the predetermined proof-load, releasing this load, and reloading the surviving specimens to failure.

(2) Post-proof fatigue life. This part of the proof-test consists of loading specimens statically to the previously selected proof load, releasing this load and then subjecting the surviving specimens to a fatigue life test at the predetermined maximum fatigue load.

### SECTION III

#### STRENGTH AND LIFE DISTRIBUTION EQUATIONS

Let  $x$  be the value of static strength. The two parameter Weibull distribution function of the static strength is then

$$F_X(x) = P[X \leq x] = 1 - \exp\left[-\left(\frac{x}{\beta}\right)^\alpha\right] \quad (1)$$

In order to compare the static strength distribution with the fatigue life distribution, we want the expression of the static strength distribution for those specimens that have strength larger than the maximum fatigue stress  $S$ . This can be obtained from the conditional probability, or.

$$\begin{aligned} F_{X,S}(x) &= P[X \leq x | X > S] \\ &= 1 - \exp\left[-\left(\frac{x}{\beta}\right)^\alpha + \left(\frac{S}{\beta}\right)^\alpha\right] \end{aligned} \quad (2)$$

Let  $n$  be the value of fatigue life, the two parameter Weibull distribution function for fatigue life is

$$F_N(n) = P[N \leq n] = 1 - \exp\left[-\left(\frac{n}{\beta_1}\right)^{\alpha_1}\right] \quad (3)$$

The strength-life equal-rank assumption stipulates that for a given specimen with static strength  $x_Y$  and fatigue life  $n_Y$ , the following relation exists

$$F_N(n_Y) = F_{X,S}(x_Y) = 1 - \gamma \quad (4)$$

where  $100\gamma$  is the percentage of the specimen that has strength larger than  $x_Y$  among those that have strength above  $S$ , and similarly, there will be  $100\gamma$  percent of specimens that have life greater than  $n_Y$ .

When we proof load a group of samples randomly selected from the population to a value of  $x_Y$ , the percentage of surviving specimens among those that have strength larger than  $S$  is

$$\gamma = \exp\left[-\left(\frac{x_Y}{\beta}\right)^\alpha + \left(\frac{S}{\beta}\right)^\alpha\right] \quad (5)$$



After a proof load  $x$ , the percentage of surviving specimens among the total population will be denoted by  $\gamma_a$ , and is given by

$$\gamma_a = \exp\left[-\left(\frac{x}{\beta}\right)^\alpha\right] \quad (6)$$

The guaranteed life at stress  $S$  is obtained from Eqs. 2, 3 and 4 by replacing  $x$  and  $n$  by  $x_Y$  and  $n_Y$ , respectively,

$$n_Y = \beta_1 \left[ \left(\frac{x_Y}{\beta}\right)^\alpha - \left(\frac{S}{\beta}\right)^\alpha \right]^{1/\alpha_1} \quad (7)$$

In studying the post proof-load static strength distribution, we formulate the "truncated" Weibull by taking proper conditional probability, or,

$$\begin{aligned} F_{X, x_Y}(x) &= P[X \leq x | X > x_Y] \\ &= 1 - \exp\left[-\left(\frac{x}{\beta}\right)^\alpha + \left(\frac{x_Y}{\beta}\right)^\alpha\right] \end{aligned} \quad (8)$$

Data of post proof-test strength will be fitted to Eq. (8) to determine the appropriate values for  $\alpha$  and  $\beta$ . When the value of  $\alpha$  and  $\beta$  for the virgin specimens are used in Eq. (8), it represents distribution of the "top  $\gamma$ -percent" of the population.

For graphical presentation of the experimental data points of strength and life cycles, we have used the median rank of each data point to represent its cumulative distribution. The median rank  $\hat{F}$  can be approximated by the formula

$$\hat{F} = \frac{j - 0.3}{m + 0.4} \quad (9)$$

where  $j$  is the failure order number, and  $m$  is the sample size. Where there are suspended, or censored, specimens, the following formula is used to calculate the order number increment of all specimens following the suspended one,

$$\Delta j = \frac{(m+1) - 1}{1+k} \quad (10)$$

where  $k$  is the number of specimens following the present suspended set.

The maximum likelihood method is used in estimating the Weibull parameters. Note that suspended or censored specimens can be handled by this method [7].

#### SECTION IV

#### RESULTS AND ANALYSIS

All test data are tabulated in a summary as displayed in Table II.

##### Static Strength Distribution

A total of 24 samples are tested here (TABLE II-A); the cumulative distribution of strength is shown in Fig. 1. It is seen that the two-parameter Weibull function fits quite well to the test data, with  $\alpha = 10.2$  and  $\beta = 1531.7$  Mpa. Note that the sample mean calculated using equation (1) is  $\bar{x} = S_m = 1462$  Mpa.

In the subsequent proof-tests, the two levels of proof-load are selected at 1290 Mpa and 1393 Mpa which correspond to  $0.88 S_m$  and  $0.95 S_m$ , respectively. From Eq. (6) we calculated the value of  $\gamma_a$  for these two proof loads as 0.16 and 0.32, respectively. These are also shown in Fig. 1 and Table III.

##### Fatigue Life Distribution

Two fatigue tests are conducted using the virgin specimens, one at the maximum fatigue load of  $0.71 S_m$ , or at 1034 Mpa; and one at  $0.81 S_m$ , or at 1179 Mpa. In the first case, a total of 130 samples are tested, with 32 failed, 35 suspended at  $10^4$  cycles, 35 suspended at  $10^5$  cycles, and 28 suspended at  $10^6$  cycles (TABLE II-B). Parameter estimation via the maximum likelihood method yields a cumulative distribution function shown in Fig. 2, with  $\alpha = 0.419$  and  $\beta = 4.59 \times 10^6$  cycles. From Eq. (5) the values of  $(1-\gamma)$  corresponding to the two proof loads are 0.145 and 0.10 respectively; the guaranteed life cycles are  $55.4 \times 10^3$  and 18 respectively. These are shown in Fig. 2 and Table III.

In the second fatigue case, a total of 25 samples are tested; of which 21 failed, 4 suspended at over  $10^6$  cycles, see TABLE II-C. The best fit to the Weibull distribution is depicted in Fig. 3. Here  $\alpha = 0.28$  and  $\beta = 59.8$  kc.



From Eq. (5), with  $S = 1179$  Mpa, the value of  $(1-\gamma)$  is 0.269 for the proof load of 1.393 Gpa, the corresponding guaranteed life is 914 cycles.

#### Post-Proof Static Strength

Test results for the post-proof static strength are tabulated in TABLE II-D and E. It is noted that in the first case, 6 samples failed during the proof-loading while 19 survived. In subsequent reloading, all 19 samples have a strength larger than the proof-load which is 1290 Mpa. The corresponding sample mean strength is 1489 Mpa. The modified Weibull distribution for the post-proof static strength of the form of Eq. (8) is used to fit the experimented data. Maximum likelihood estimation of parameter gives the results of  $\alpha = 12$  and  $\beta = 1510$  Mpa, as shown in Fig. 4. This distribution is compared with the distribution of the top 84% data of the virgin specimens (those larger than 1290 Mpa in TABLE II-A, sample mean is 1510 Mpa). It is seen that these two distributions are practically identical, although there exhibits a tendency of reduced scatter for the post-proof strength. This can be seen more clearly by comparing their respective density functions, Fig. 5.

In the second case, a total of 25 samples are tested; of which 8 failed during proof-loading and 16 survived. A Weibull fit of the post-proof strength distribution is shown in Fig. 6. The corresponding sample mean is 1572 Mpa. This is again compared with the distribution of the top 68% data of the virgin specimens (those larger than 1393 Mpa, sample mean is 1545 Mpa.) Here the post-proof distribution has  $\alpha = 23$  and  $\beta = 1572$  Mpa. Clearly, proof-loading reduces the scatter, while it does not affect, appreciably, the sample mean strength. The respective distribution density functions are shown in Fig. 7 with a pronounced reduction in scatter exhibited by the post-proof strength.

Figure 8 gives a summary of the data points and various sample mean strength values for the two post-proof static strength cases.

#### Post-Proof Fatigue Life

Results of the post-proof fatigue life tests are tabulated in TABLE II-F, G, and H. The first case pertains to proof-loading the specimens to 1290 Mpa ( $88\% S_m$ ) and then subjects the surviving specimens to a fatigue test at the maximum fatigue load of 1034 Mpa ( $0.71 S_m$ ). Here, a total of 25 samples are tested, of which 4 failed the proof-load, 3 failed during fatigue test and 18 suspended at 70 kc. An S-N scan for the test data is shown in Fig. 9.

From Table III it can be seen that the 1290 Mpa proof load screen out 14.5% of the low life specimens. From Eq. (7), we obtained a guaranteed life of 55.4 kc. In Fig. 9, it is seen that one sample failed before reaching a life of 55.4 kc, two samples failed at about 55.4 kc while 18 others have a life greater than this.

In the second case, 25 samples are proof-loaded again to 1290 Mpa. Here, 4 failed the proof-load, 18 failed during fatigue at a maximum fatigue load of 1179 Mpa ( $0.81 S_m$ ) while 3 samples are suspended at  $10^6$  cycles. Fig. 10 shows the S-N scan for these data. Again, if a minimum life can be inferred from the static strength distribution and the fatigue life distribution at the corresponding load level, the minimum life expected for this case should be 17.8 cycles. From the test data, all the 21 specimens exceed the minimum life.

Similarly, a total of 25 samples are tested in the third case. The proof-load here is 1393 Mpa ( $0.95 S_m$ ) and the maximum fatigue load is 1179 Mpa ( $0.81 S_m$ ). In this case, 9 samples failed the proof-load, 14 failed during fatigue and 2 suspended at  $10^6$  cycles. The inferred minimum life is 914 cycles; but two samples out of 16 failed to meet this expectation, Fig. 11.

## SECTION V

### DISCUSSIONS

As it has been stated earlier, most current fatigue analyses for composite laminates are based implicitly or explicitly upon the so-called equal-rank assumption. This, it is important to verify this assumption with sufficient test data and test cases. In the present study, the effects of proof-test on static strength and fatigue life are investigated for unidirectional graphite-epoxy laminates. From these results the validity of the equal rank assumption can be evaluated against experiment.

For the post-proof static strength, the results indicate that proof-load does not change the essential features in the static strength. However, the post-proof specimens generally have a smaller scatter in their strength distribution as compared to the strength distribution of the corresponding top percentile of the virgin specimens. Moreover, all specimens that survived the proof-load show a post-proof strength larger than the proof-load. The results thus indicate a 100% probability that proof-load can guarantee a minimum strength. In a different viewpoint, the results also indicate that proof-load does not alter the ranking in the strength distribution.

The probability of guaranteeing a minimum life after proof-load is slightly less assuring; the results show, however, that the chance is 90% or better. Note again that two conditions must be met if a minimum life can be guaranteed after proof-loading; first, the proof-load must not cause damage so as to degrade the fatigue property; and secondly, the "equal-rank" assumption must be valid. In view of the overall results, it may be stated that proof-load will alter slightly the strength and fatigue properties of the specimen, only if the proof-load is high. The small alteration in property is due to



proof-loading induced damage. On the other hand, the evidence obtained in the test results indicates that the equal rank assumption is generally valid.

The specimens used here are all unidirectional composites. It is not clear, however, whether the concept of proof-test is equally applicable in composite laminates of different fiber orientations and stacking sequences. Such a question clearly needs further study; and its implication could be of considerable practical importance.

# Nomenclature

$\hat{F}$	Median rank
$F_X(x)$	Cumulative distribution function for random variable X
$F_{X,S}(x)$	Conditional cumulative distribution function for random variable X
j	Failure order number
k	Number of specimens following a suspended set
m	Sample size
n	Value of fatigue life
$n_Y$	Value of guaranteed fatigue life
N	Random variable of fatigue life
$P[X \leq x]$	Cumulative distribution function for random variable X
$P[X \leq x   X > S]$	Conditional cumulative distribution function for random variable X
S	Maximum stress applied in fatigue cycling
$S_m$	Sample mean
$\bar{x}$	Sample mean
x	Value of static strength
$x_Y$	Value of proof load
X	Random variable of static strength
$\alpha$	Weibull shape parameter for static strength
$\alpha_1$	Weibull shape parameter for fatigue life
$\beta$	Weibull scale parameter for static strength
$\beta_1$	Weibull scale parameter for fatigue life
$\gamma$	Percentage of surviving specimens
$\gamma_a$	Percentage of specimens surviving a proof load among the total population
$\Delta j$	Failure order number increment of all specimens following a suspended set

**TABLE 1**  
**Number of Specimens Tested**

	Static Strength	Fatigue Life	
		Max. Stress = $0.71 S_m$ (1.034 Gpa)	Max. Stress = $0.81 S_m$ (1.179 Gpa)
Base-Line Data (No Proof-load)	24 (A)*	130 (B)	25 (C)
Proof-load to $0.88 S_m$ (1.29 Gpa)	25 (D)	25 (F)	25 (G)
Proof-load to $0.95 S_m$ (1.39 Gpa)	25 (E)		25 (H)

\* Letter in parenthesis gives Section in Table II where details are given.

$S_m$  = sample mean of Static Strength



TABLE 2  
Summary of Test Results

**A. Static Strength of Virgin Specimens.**  
24 specimens failed. Unit in Mpa.

1096	1339	1445	1534	1634
1221	1347	1471	1544	1689
1227	1416	1476	1573	1729
1287	1421	1481	1576	1760
1305	1429	1491	1584	

sample mean strength  $S_m = 1462$

**B. Fatigue Life of Virgin Specimens.** Cycles; max. stress = 1034 Mpa (71%  $S_m$ ),  
R = 0.1, f = 9.5 Hz, 130 specimens, 98 suspended, 32 failed.

1(2)**	2330	14260	95606	441030
29	8350	27300	96310	531170
450	9550	37770	96360	844080
844	10000*(35)	57450	100000*(35)	1000000*(28)
860	10810	68517	222220	1049160
1770	12781	76890	327580	1874600
2315	13261	86580	398480	

\* Specimen suspended before fatigue failure

\*\* Failed before max. load is reached; not included in life distribution calculation.

**C. Fatigue Life of Virgin Specimens.** Cycles; max. stress = 1179 Mpa (81%  $S_m$ ),  
R = 0.1, f = 9.5 Hz, 25 specimens, 4 suspended, 21 failed.

1**				
30	288	5984	15754	1000000*
69	380	8609	18995	1000000*
90	1570	11362	22570	1066620*
260	3269	12119	97009	3302720*
286	5653	15520	149356	

\* Specimen suspended before fatigue failure

\*\* Failed before max. load is reached; not included in life distribution calculation.

**D. Static Strength of Proof-Loaded Specimens.** Proof-Load = 1290 Mpa (88%  $S_m$ ),  
unit in Mpa.; 25 specimens, 6 failed during proof loading.

1041*	1289*	1407	1482	1551
1207*	1358	1441	1482	1558
1241*	1358	1462	1531	1593
1262*	1400	1469	1531	1600
1269*	1407	1476	1538	1744

\* failure during proof-load.

- E. Static Strength of Proof-Loaded Specimens. Proof Load = 1393 Mpa (95%  $S_m$ ); unit in Mpa; 25 specimens, 8 failed during proof loading.

1041*	1310*	1517	1545	1620
1151*	1338*	1524	1545	1655 (2)
1227*	1365*	1524	1572	1669
1255*	1447	1538	1593	1682
1303*	1476	1545	1600	

\* failure during proof-load.

- F. Fatigue Life of Proof-Loaded Specimens. Cycles, proof-load = 1290 Mpa (88%  $S_m$ ); max. Fatigue Stress = 1034 Mpa, (0.71  $S_m$ ); 25 specimens, 3 failed during proof-load, 4 failed during fatigue load, 18 suspended at 70,000 cycles.

0(1117 Mpa)*	20481
0(1172 Mpa)*	39000
0(1269 Mpa)*	39643
289**	70000(18)***

\* failure during proof-load

\*\* specimen showed severe damage after proof-load.

\*\*\* 18 specimens suspended at 70000 cycles.

- G. Fatigue Life of Proof-Loaded Specimens. Cycles; proof load = 1290 Mpa (88%  $S_m$ ); max. Fatigue stress = 1179 Mpa (0.81  $S_m$ ); 25 specimens, 4 failed during proof-load, 18 failed during fatigue load, 3 suspended at 1,000,000 cycles.

0(1241 Mpa)*	1060	22860
0(1241 Mpa)*	1200	29440
0(1248 Mpa)*	1870	68010
0(1261 Mpa)*	2510	368280
100	3210	782120
150	5430	964760
200	9050	1000000(3)**
960	16910	

\* failure during proof-load

\*\* 3 specimens suspended at  $10^6$  cycles.

- H. Fatigue Life of Proof-Loaded Specimens. Cycles; proof-load = 1393 Mpa (95%  $S_m$ ); max. fatigue stress = 1179 Mpa (0.81  $S_m$ ); 25 specimens, 9 failed during proof-load, 14 failed during fatigue load, 2 suspended at 1,000,000 cycles

0(1255 Mpa)*	0(1379 Mpa)*		
0(1255 Mpa)*	50	13510	1000000(2)**
0(1289 Mpa)*	470	13770	
0(1289 Mpa)*	2340	16770	
0(1289 Mpa)*	2370	22980	
0(1303 Mpa)*	4210	142870	
0(1379 Mpa)*	7230	167760	
0(1379 Mpa)*	8980	866070	

\* failure during proof-load

\*\* 2 specimens suspended at  $10^6$  cycles.

TABLE 3  
Parameters in Guaranteed Fatigue Life  
after Proof-loading

Fatigue Load, S							
Proof Load	.71 S <sub>m</sub>				.81 S <sub>m</sub>		
	Percent of Failure Among All Specimens $1 - \gamma_a$	Percent of Failure Among $X \geq S$	Guaranteed Life, Cycles	Experimental Percentage Exceeding Min Life	Percent of Failure Among $X \geq S$	Guaranteed Life, Cycles	Experimental Percentage Exceeding Min Life
$x_\gamma$	$1 - \gamma_a$	$1 - \gamma$	$n_\gamma$		$1 - \gamma$	$n_\gamma$	
.88 S <sub>m</sub> (1.290 Gpa)	16	14.5	55350	85.7	10	17.8	100
.95 S <sub>m</sub> (1.393 Gpa)	32	-	-	-	26.9	914	87.5



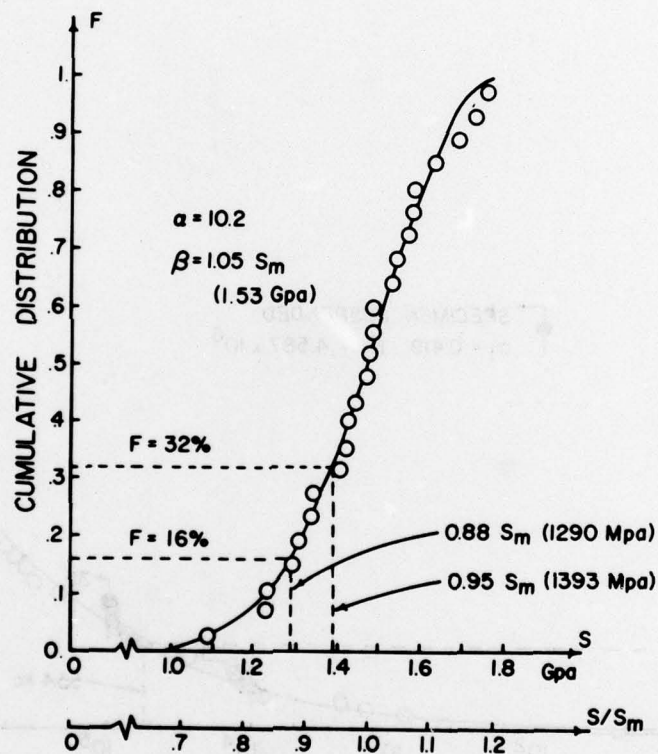


Figure 1. Cumulative distribution of static strength of virgin specimens

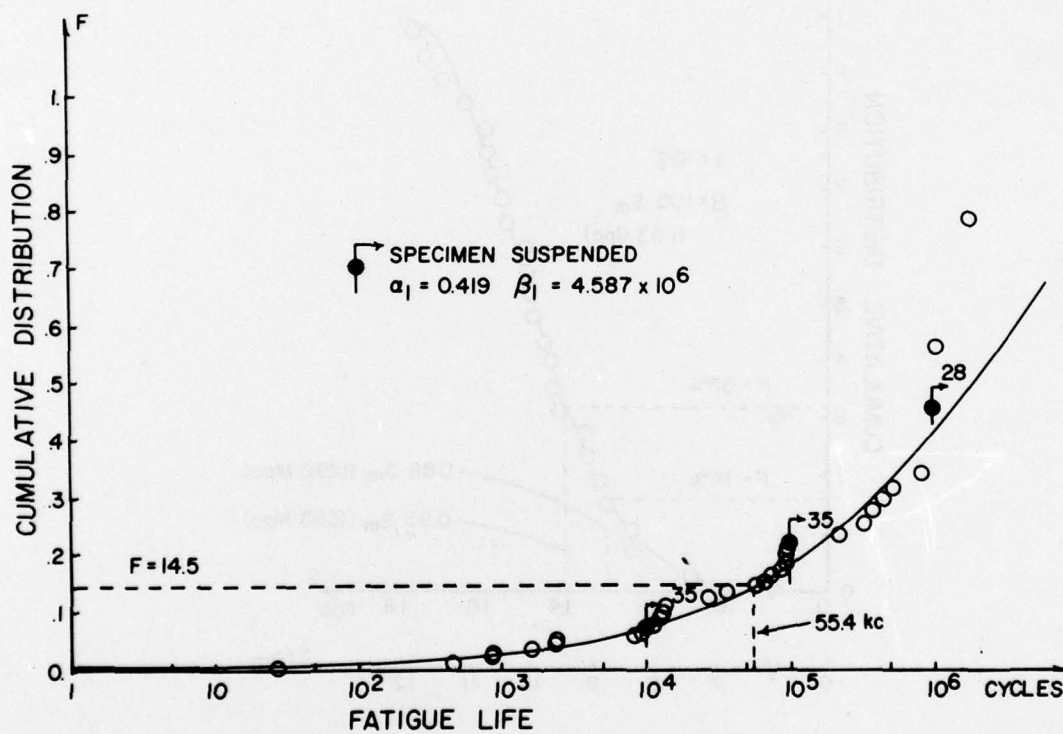


Figure 2. Cumulative distribution of fatigue life;  
 virgin specimens; fatigue load =  $0.71 S_m$  (1.034 Gpa)

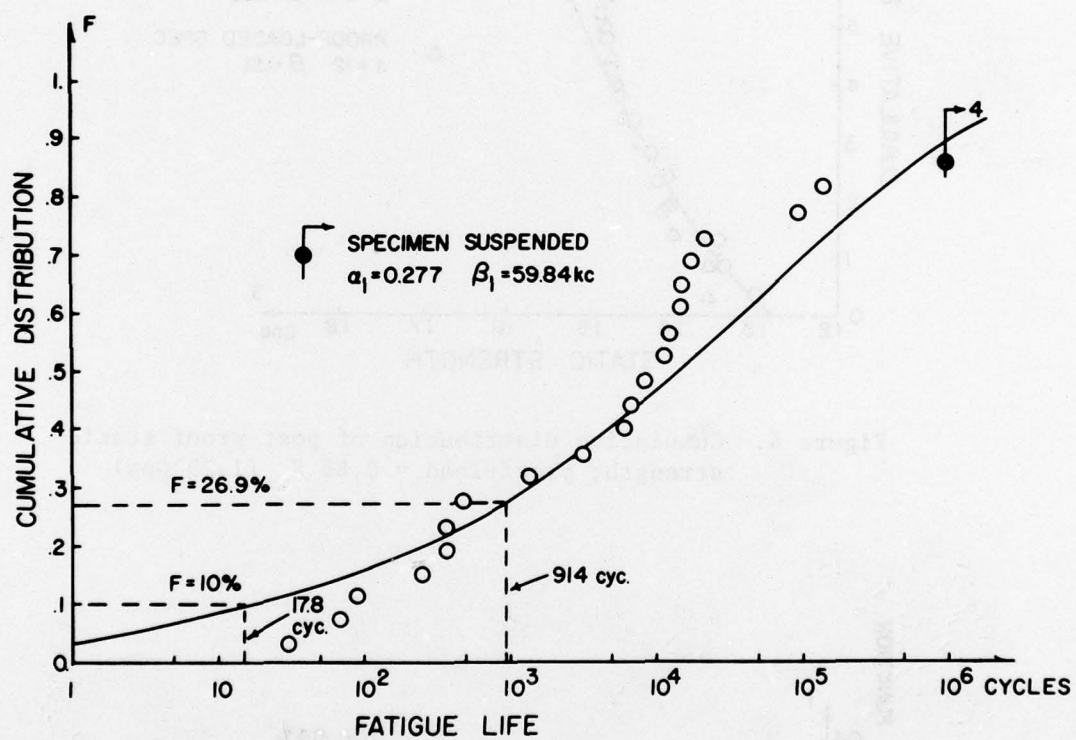


Figure 3. Cumulative distribution of fatigue life; virgin specimens; fatigue load =  $0.81 S_m$  (1.179 Gpa)



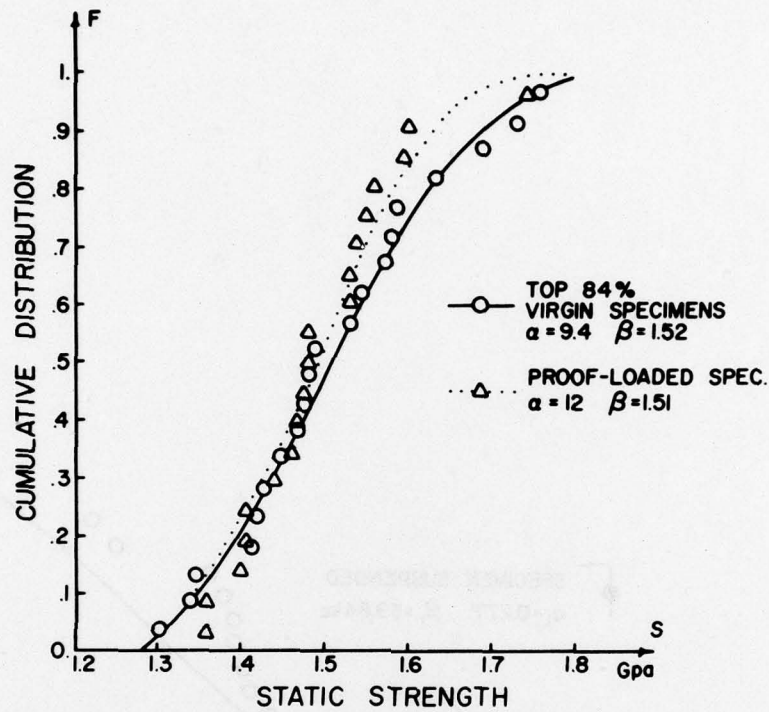


Figure 4. Cumulative distribution of post-proof static strength; proof-load =  $0.88 S_m$  (1.29 GPa)

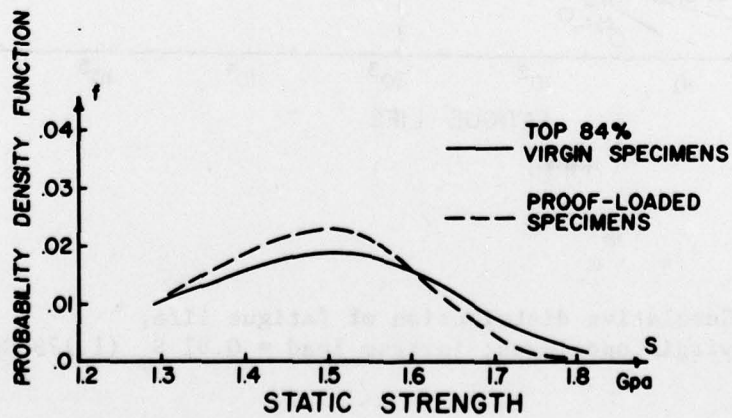


Figure 5. Weibull density function for post-proof static strength; proof-load =  $0.88 S_m$  (1.29 GPa)

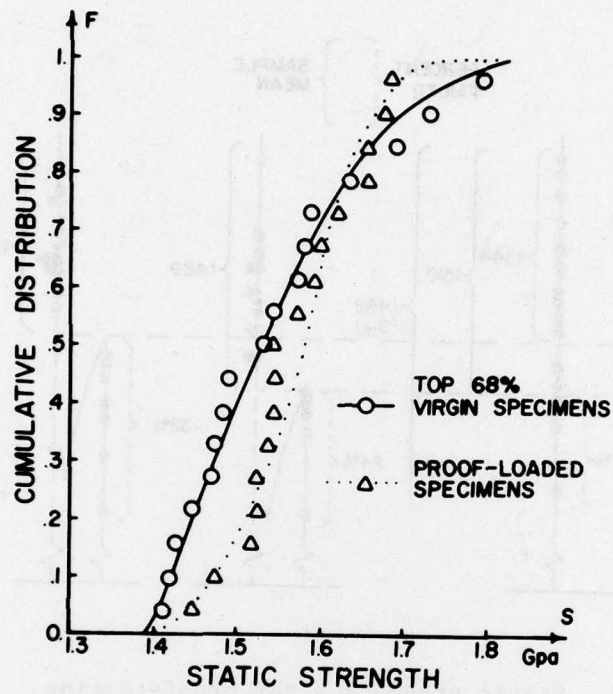


Figure 6. Cumulative distribution of post-proof static strength; proof-load =  $0.95 S_m$  (1.393 Gpa)

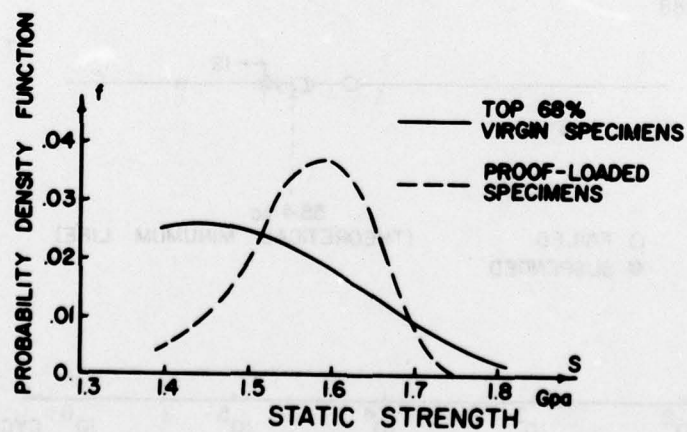


Figure 7. Weibull density function of post-proof static strength; proof-load =  $0.95 S_m$  (1.393 Gpa)

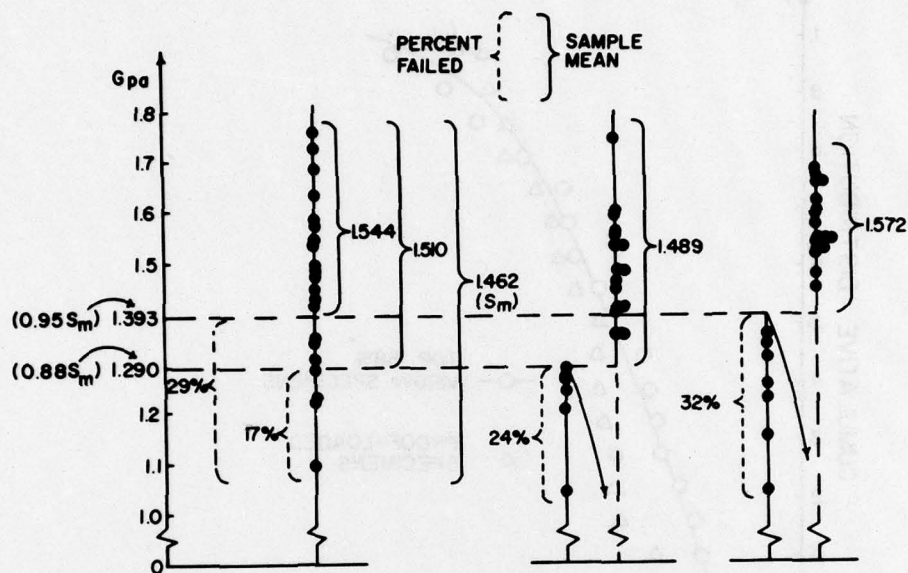


Figure 8. Static strength after proof-testing

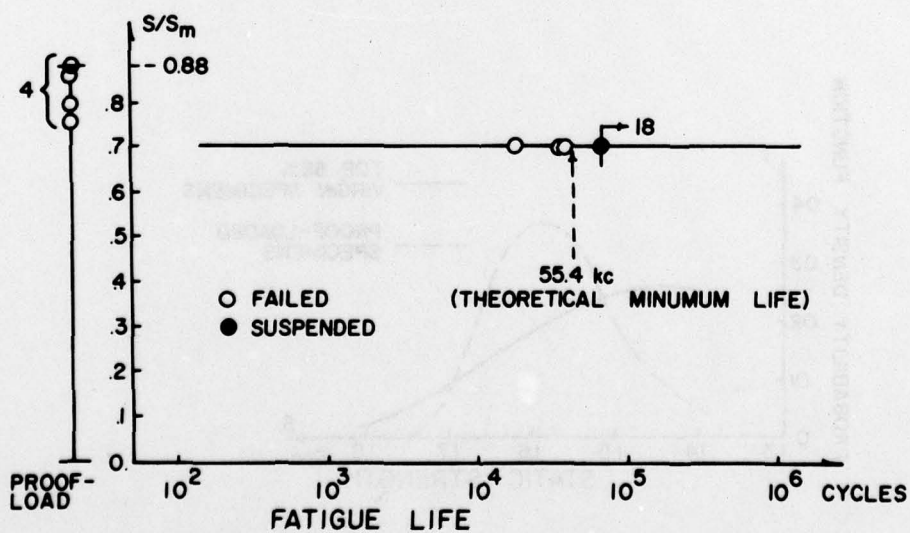


Figure 9. S-N scan for fatigue of post-proof specimens; fatigue stress =  $0.71 S_m$  (1.034 Gpa); proof-load =  $0.88 S_m$  (1.29 Gpa)



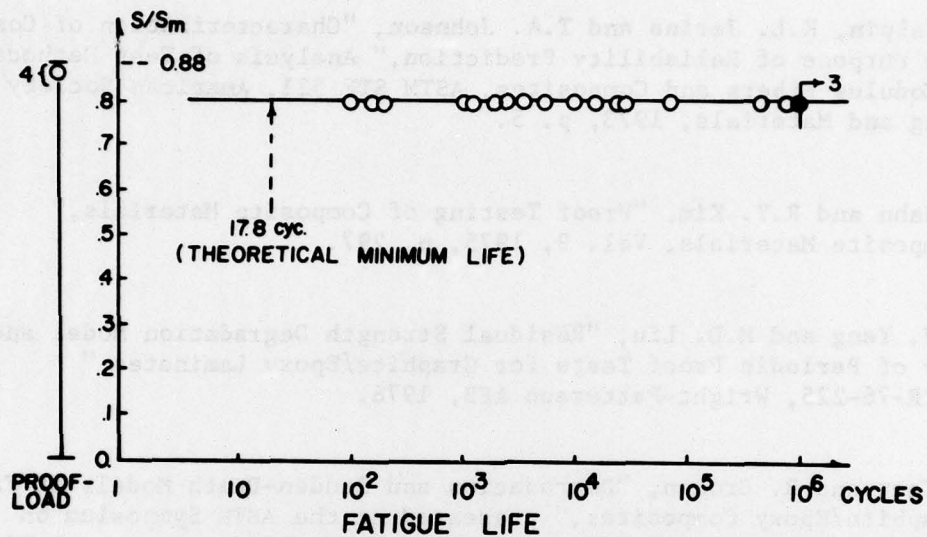


Figure 10. S-N scan for fatigue of post-proof specimens; fatigue stress =  $0.81 S_m$  (1.179 Gpa), proof-load =  $0.88 S_m$  (1.29 Gpa)

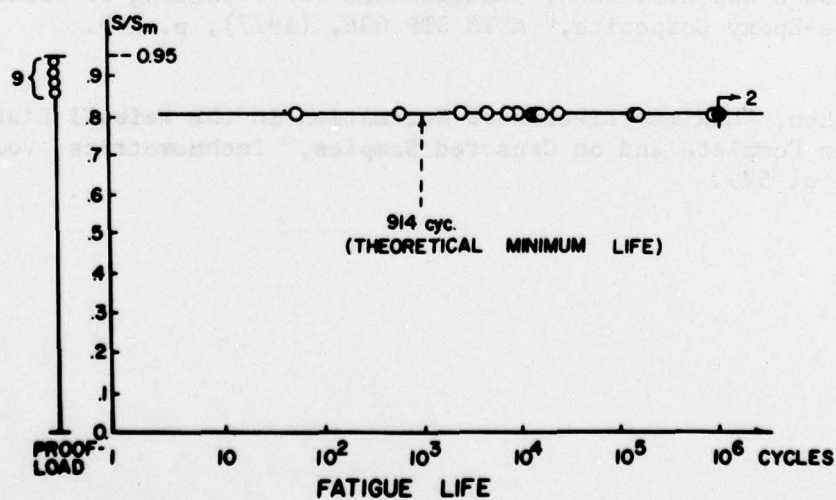


Figure 11. S-N scan for fatigue of post-proof specimens; fatigue stress =  $0.81 S_m$  (1.179 Gpa); proof-load =  $0.95 S_m$  (1.393 Gpa)

#### REFERENCES

1. J.C. Halpin, K.L. Jerina and T.A. Johnson, "Characterization of Composites of the Purpose of Reliability Prediction," Analysis of Test Methods for High Modulus Fibers and Composites, ASTM STP 521, American Society for Testing and Materials, 1973, p. 5.
2. H.T. Hahn and R.Y. Kim, "Proof Testing of Composite Materials," J. Composite Materials, Vol. 9, 1975, p. 297.
3. Jann N. Yang and M.D. Liu, "Residual Strength Degradation Model and Theory of Periodic Proof Tests for Graphite/Epoxy Laminates," AFML-TR-76-225, Wright-Patterson AFB, 1976.
4. P.C. Chou and R. Croman, "Degradation and Sudden-Death Models of Fatigue of Graphite/Epoxy Composites," presented at the ASTM Symposium on Composite Materials; Testing and Design, New Orleans, La. March 20, 1978.
5. P.C. Chou and R. Croman, "Residual Strength in Fatigue Based on the Strength-Life Equal Rank Assumption," Jour. Composite Matls. Vol. 12 (1978), p. 177.
6. J. Awerbuch and H.T. Hahn, "Fatigue and Proof-Testing of Unidirectional Graphite-Epoxy Composite," ASTM STP 636, (1977), p. 248.
7. A.C. Cohen, "Maximum Likelihood Estimation in the Weibull Distribution Based on Complete and on Censored Samples," Technometrics, Vol. 7 (1965), p. 579.